

Memory Management Schemes for Radiosity Computation in Complex Environments

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Abstract

Hierarchical radiosity is a very demanding process in terms of computation time and memory resources even for scenes of moderate complexity. To handle complex environments, not fitting in memory, new solutions have to be devised. One solution is to partition the scene into subsets of polygons (3D cells or clusters) and to maintain in memory only some of them. The radiosity computation is performed only for this resident subset which changes during the resolution process. This change entails many read and write operations from or onto the disk. These disk transfers must be ordered to make the radiosity algorithms tractable. We propose in this paper different ordering strategies which can be seen as complementary to those devised by Teller [1].

Keywords :

Radiosity method, ordering, complex scenes, architectural environments.

1 Introduction

Hierarchical radiosity provides high level realistic images at the expense of important computation and memory resources, even for scenes of moderate complexity. This is due to the need of meshing surfaces into elements and linking these latter one to another. In other words, the number of surface elements and links increases drastically with the precision required. This is why performing lighting simulation becomes tricky for complex scenes such as buildings containing millions of geometric primitives. Indeed, visibility relationships between surface elements have to be performed, which entails a very important computing time. However, many visibility computations are useless such as those between a polygon and some objects occluded by a large polygon like a wall. This problem has already been addressed in [2, 3, 4, 5] where the authors propose to partition the scene into subsets of polygons (called *cells*) before

performing radiosity computations whether sequentially [1] or in parallel [6].

Another problem is due to the fact that the amount of data needed for radiosity computations (for very complex scenes) is so large that they cannot fit in memory. One solution to this problem is to partition the scene into cells so that only a subset of cells is maintained in memory for illumination computations. This is made possible with the help of visibility relationships between these cells. Indeed, a polygon in a subset C shoots/gathers energy to/from all the objects lying in C as well as those visible through the holes (also termed *portals*) in its boundary like windows or doors for buildings interiors. Thus, one needs to load in memory the cells (stored on the disk) concerned with the radiosity computations, to withdraw the non-necessary cells from memory and move them back to the disk. This operation requires many disk transfers which are very time consuming. These transfers must be ordered so as to reduce expensive read and write operations (from/onto the disk or memory).

In this paper, several ordering strategies, different from the ones described in [1], are proposed. These strategies make use of the cells resulting from the scene partitioning as well as a graph expressing visibility between these cells.

The organization of this paper is as follows. Previous works are first summarized then followed by an overview of our algorithm. Next, our ordering strategies are described in detail. Finally, results of experiments are provided to demonstrate the usefulness of these kinds of strategy.

2 Previous Works

A few works address the problem of global illumination computation for complex scenes containing millions of objects. To our knowledge, Teller [1] is one of the first author who proposed an algorithm for solving this problem. His objective is to use radiosity for global illumination computations and a gathering scheme for the resolution. To this end, his approach consists in subdividing the scene into 3D cells with the help of a BSP-based partitioning technique. This latter allows to efficiently compute visibility between clusters. A datastructure is associated with each visibility

relationship between two visible clusters. This datastructure contains: a list of blockers, a list of links between the surfaces of both clusters, convex hulls of clusters, etc. These data enable each cluster to gather energy from the other clusters visible to it.

The datastructures resulting from the BSP partitioning, clustering and visibility calculations are stored on disk. Only a small portion of these data resides in memory for radiosity computations. As the algorithm utilizes gathering, it gathers energy impinging on each cluster. To this end, it loads in memory a receiving cluster R as well as one of the visible emitting cluster S . Afterwards R gathers energy from S . This operation is repeated for all the other clusters S visible from R . Each source cluster S modified by the gathering operation (creation of new surface elements) is saved on disk. Despite the use of the disk cache, these disk transfers remain too high. This is why Teller proposes several heuristics for efficiently ordering these disk transfers so that the contents of the cache changes slowly. These heuristics are:

(a) **Random order:** Gathering is randomly performed; (b) **Model order:** Clusters gather energy in the order in which they were instantiated by the modeler; (c) **Source order:** Select the receiving cluster which has most often acted as a source; (d) **Cell order:** Schedules clusters by traversing the cells of the BSP tree. In this way consecutive cells are chosen by selecting the neighbour cell seen from the largest portal. This approach exploits the visibility coherence of clusters due to proximity and local intervisibility.

From the experiments done by Teller, the fourth approach which exploits the result of the BSP partitioning method seems giving the best results.

A parallel algorithm for radiosity computation in large environments has been proposed by Funkhouser [6]. This algorithm makes use of the same datastructures as those proposed by Teller. The scene is organized in groups of clusters which are distributed by the host processor to the slave processors. Each slave processor is in charge of computing the radiosity solution for the group it is responsible for. Funkhouser proposes two different strategies for load balancing giving interesting speed-ups.

Note that, once global illumination computation has been completed for complex environments with the methods described above, rendering techniques can be applied to interactively walk through these environments [7] [8] [9] and [10]. However, this topic is out of the scope of this paper.

3 Overview

Our algorithm allows radiosity computation for complex scenes such as buildings interiors made up of polygonal objects. A preprocessing step partitions the scene into subsets

called *cells*, and a graph expressing visibility relationships between these cells is built [5]. In this visibility graph, a vertex represents a cell and an edge between two vertices A and B means that objects within A see other objects within B and vice versa. This visibility graph together with the obtained cells are used by our ordering strategies for radiosity computations. Our radiosity method relies on projection techniques with wavelets as basis functions and offers two resolution schemes: gathering and shooting. More precisely, our wavelet radiosity is hierarchical and the degree of the wavelet basis functions varies from 0 to 3, which corresponds to vanishing moments ranging from 1 to 4. In our implementation, we use shooting because of its ability to quickly provide useful images at the early stages of the resolution and to interactively walk through the scene while the radiosity computations are being performed. This is made possible even for complex environments containing thousands of polygons thanks to the visibility graph.

The main contribution of our work is concerned with new ordering strategies. Ordering has already been addressed by Teller [1]. As seen in the previous section, his approach is based on a precise but complex and very costly visibility preprocessing involving clusters, tubes, blockers, etc. In addition, in [1] the ordering techniques consist in loading into memory a cluster, the clusters visible from it as well as potential blockers. Unlike Teller, our ordering strategies rely on very simple visibility computations between cells instead of clusters. Indeed, the method we use to compute visibility between two cells is due to Airey [2] and consists in tracing rays through the portals separating these cells.

Another difference with Teller's work is that our ordering algorithms allow to load into memory a cell C (instead of a cluster) together with those cells C_i visible from C . Once these cells have been loaded into memory, the surface elements within C shoot their energy towards the ones within the cells C_i .

As said in the previous section, Teller's method relies on an optimal use of the disk cache. However with this approach the user has no control on the cache, which is a serious problem in case of multiuser computer. For example one cannot impose maintaining in memory certain data necessary for the current radiosity computation. This is why one contribution of our work is to manage the memory (destinated to store the data corresponding to the cells) as a software cache. The management of this software cache is handled by our ordering strategies which predict for the short, medium and long range terms the disk transfer costs that determine the choice of the cells that will shoot their energy. These strategies rely on discrete optimisation techniques that explicitly account for the actual disk transfers.

Our ordering strategies are part of a global illumination software package composed of different modules: ordering

module, radiosity calculation module and visibility graph module.

Let us recall that our resolution scheme is shooting. A cell whose surfaces are selected for shooting their energy will be called *shooting cell* from now on. The role of the ordering module is to select a shooting cell as well as those visible from it, with the help of one of the ordering strategies (which will be described later on) depending on the cells already in memory and the visibility graph. A linear array, termed *Cell_In_Memory[]* points out whether a cell is already in memory or not and if it will be used by the radiosity module. Note that a cell is used for radiosity computations if and only if it has been selected by the ordering module. Once the cells have been selected by the ordering module, the radiosity module is invoked.

Our global illumination algorithm is given in figure 1. In one *iteration* (called ITERATION in this algorithm), all the cells are in turn selected as shooting cell.

The function *Choose_Cell()* is no more than the ordering module. It corresponds to the implementation of one of the ordering strategies we have developed. As for the procedure *Shoot_From_Cell()*, it allows each surface of the selected cell *C* to shoot its energy towards the surfaces within *C* as well as those belonging to the cells visible to *C*.

The next section describes in detail our ordering strategies.

4 Ordering

4.1 Principle

Let us recall that the objective of this work is to compute global illumination in very large environments with the radiosity method. As in [1], to meet this goal, the scene is partitioned into 3D cells and a graph expressing visibility between these cells is built. This graph allows to maintain a subset of cells in memory for which the radiosity computation is performed. Radiosity computation consists in choosing a set of cells, selecting one of them for shooting its energy to the others belonging to the same set. This process is repeated for other sets of cells till convergence. It needs to load in memory the cells (stored on disk) concerned with the radiosity computations, to withdraw certain non-necessary cells from memory and move them back to the disk. This operation requires many disk accesses, which is very costly in terms of computation time. These disk accesses correspond to the transfer of cells between the memory and the disk. These transfers must be ordered so as to reduce expensive read and write operations (from/onto the disk or memory). To be effective, an ordering algorithm must schedule successive shootings so as to minimize disk accesses, while maintaining rapid convergence properties. This problem is

equivalent to finding a solution to the traveling salesman problem which is more complex in our case because the database changes dynamically (creation of new links, new elements due to the hierarchical radiosity approach).

Our ordering strategies operate according to the algorithm given in figure 2. Whenever a cell is chosen for shooting its energy, it has to be in memory as well as those visible to it. In case one of these cells is not already in memory, it is read from the disk, implying then memory/disk transfers whose cost is evaluated in terms of input surfaces or cells depending on the used ordering strategy. This cost is evaluated by the function *evaluate_cost()*.

Update() is a procedure which sets to TRUE the fields *In_Memory* and *To_Be_Used* of the datastructures *CELL_FLAGS* (see figure 1) associated with the cells in VS (see figure 2). It also sets to FALSE only the field *To_Be_Used* associated with cells not visible from the selected shooting cell but already in memory.

In our implementation, we have limited to a value M_{limit} the size of the memory used for storing the cells, which corresponds to a certain number of cells that varies during the radiosity computation process. We have chosen a value proportional to the maximum number of input surfaces that can be stored in memory. We have preferred to make the limitations of memory size rely on the number of surfaces rather than the actual memory size (that closely depend on the number of surface elements and links). Indeed, since the number of surface elements and links has a complexity of $O(n \log n)$, n being the number of input surfaces, we can a priori estimate M_{limit} , which is more reliable. This estimation prevents from exceeding M_{limit} . For the same reason, for some of our ordering strategies the disk transfers cost is evaluated in terms of number of input surfaces. For the other strategies this cost corresponds to the number of cells to be transferred from and onto disk.

4.2 Strategies

4.2.1 Random method

This method will serve as reference to the other strategies described hereafter. The shooting cell is randomly chosen then loaded into memory together with the cells that are visible from it. This strategy does not account for the cost of disk transfers.

4.2.2 Greedy method *S*

The objective is to reduce the cost of disk transfers expressed in terms of number of input surfaces. Each cell is considered as a potential shooting cell and its associated cost is evaluated. This cost corresponds to the possible transfer from the disk of this cell and its associated visible

set. The cell with minimum cost is selected as the actual shooting cell.

4.2.3 Greedy method C

Due to the above remark, this strategy is similar to the previous one except the fact that the cost is evaluated in terms of number of cells.

4.2.4 The traveling salesman strategy

As ordering is equivalent to the traveling salesman problem, we propose an algorithm for solving this problem.

Using the visibility graph we build another graph which is oriented and valued. In this new graph OVG a vertex corresponds to a cell and with each pair of cells $(C1, C2)$ is associated an oriented edge. The value attached to an edge $(C1, C2)$ is the cost due to the cell $C2$ (in terms of number of surfaces) in case $C2$ would be chosen as the next shooting cell just after $C1$. Note that the value of an edge corresponds only to the cost of reading cells from disk. This is due to the fact that since the construction of OVG is performed before the radiosity computation, we cannot foresee which cells, already in memory, have to be moved back to disk.

The graph OVG is used by the traveling salesman strategy (for further implementation details, see [11]).

4.2.5 Backtracking S

Recall that the greedy algorithms select one shooting cell at each step of the resolution process. As this choice cannot be reconsidered, this can make the ordering less efficient. One solution is to select, among all the lists of N successive shooting cells, the one entailing the lower cost. This amounts to predict, at one go, N best shooting cells. This solution is provided by the backtracking algorithm described hereafter.

Given a set of cells $CELL$, the objective is to find out a list of N cells which will be successively selected as shooting cell. To this end we select a first cell C_0 as shooting cell. Next we determine the subsequent shooting cells: $C_1 \in \{CELL - C_0\}$, $C_2 \in \{CELL - \{C_0, C_1\}\}$, ... , $C_{N-1} \in \{CELL - \{C_0, C_1, \dots, C_{N-2}\}\}$. All the possibilities for choosing these N cells are represented by a tree (having a depth N) whose root is C_0 and vertices the cells. In this tree the value associated with each edge (C_i, C_j) represents the cost due to the disk transfers when C_j is selected as shooting cell just after C_i . The backtracking strategy consists in traversing this tree in order to determine the path of depth N and of minimum cost. The cost is evaluated in terms of number of input surfaces.

4.2.6 Backtracking C

This strategy is similar to the previous one except the cost is evaluated in terms of number of cells to transfer from the disk.

4.2.7 Max energy

The objective of this strategy is to speed-up the convergence to the radiosity solution. With this aim in view, the cell with maximum unshot energy is selected as shooting cell. Note that this method does not worry about the cost due to disk transfers.

5 Implementation and results

5.1 Implementation

Files structure

In a preprocessing step the scene is partitioned into 3D cells. Each cell is stored on disk in a file containing a list of input surfaces, reflectances, spectral power and intensity distributions of the light sources. The visibility graph is also saved in a file containing the description of each cell. Indeed, in this file a cell is described by the following data: (i) the name of the file containing the cell, (ii) the number of the cell, (iii) the list of cells visible from it (the visible set VS).

Partitioning into cells

Only building interiors are considered as scenes. These latter are modeled with planar convex polygons with the help of a modeler we have developed specially for complex environments. This modeler is capable of creating different kinds of geometric entities such as polygonal objects, walls, ceilings, doors, windows, etc. It also allows the duplication, removal, geometric transformation of all these entities.

To partition the scene (not necessary axial) into 3D cells, we have used the method described in [5].

Radiosity computation

The radiosity computation starts once this partitioning has been completed. During this computation the contents of these cells is modified to include the created surface elements and associated links. Whenever a modified cell is saved on disk its corresponding file is also modified to contain the new data.

Each surface element is identified by its number and the number of the cell containing it. This identification makes possible read and write (from or onto disk) of links between surface elements belonging to different cells.

An option is offered to the user for interactively rendering the scene at the intermediate stages of the resolution while pursuing radiosity computations. To do this, two processes are resident in memory: the first one performs the radiosity computations while the second is in charge of rendering. As soon as a cell has completed its shooting a *UNIX signal* is sent to the rendering process which renders the modified scene.

Another possibility of our global illumination algorithm (figure 1) is to stop the calculations after a number of *iterations* fixed by the user and to save the cells in memory which are in a certain state. Later on, we can restart, from this state, the program to continue the radiosity calculations till convergence.

Memory management

Disk transfers involve memory allocations and releases which are managed by the routines *malloc()* and *free()* of the C library *libc.so*. However, after several experiments we have ascertained many swaps whereas the data resident in memory did not require any swap. This is due to the fact that the fragmentation of the main memory is inefficiently managed by these routines. Consequently, we have decided to manage memory by ourselves.

5.2 Results

The seven ordering strategies we have explained in the previous section have been applied to different scenes. This section provides some results only for one scene which is a building composed of three floors. It is made up of 57,786 input surfaces among which about 2,000 are light sources. This scene has been partitioned into 615 cells. M_{limit} has been set to 250Mo which corresponds to the storage of 21 cells and about 2,000 input surfaces. The number of resulting surface elements is about 360,000 while the number of links (between surface elements) is equal to 3.7 millions. The storage of all these datastructures would need about 3Go which is not offered by most of the computers even with a virtual memory.

The results obtained with the different strategies are given by figures 4 and 5 as well as by table 1.

In table 1 the computing times corresponding to the ordering strategies are given. We can see that the four best strategies, in terms of computing time, are: Traveling Salesman, Greedy C, Backtracking C and Greedy S.

The figures 4, 5 provide some results, illustrated by three plots, for each of these four best strategies. The first one (red plot) provides the number of surfaces resident in memory at the i^{th} iteration (recall that in one iteration all the cells are in turn selected as shooting cell once only). The second gives the number of surfaces belonging to the current shooting cell and to the cells visible from it (green plot).

The third one gives the cost of disk transfers expressed in terms of number of input surfaces for the i^{th} selected shooting cell (blue plot). For a reason of clarity, these plots correspond to only one iteration. Note that the shape of these plots does not vary from one iteration to another.

For each strategy the number of disk transfers, expressed in terms of input surfaces, is given by table 2.

Many conclusions can be drawn from these results. First of all, the *Random* and *Max_Energy* methods involve a more important amount of disk transfers. This result is foreseeable since these latter are not accounted for by these methods. Moreover, the strategies based on the number of transferred cells are more efficient than those based on the number of transferred surfaces. This can be explained as follows. When evaluating the disk transfer cost in terms of number of surfaces, the cells made up of many surfaces are selected only at the end of each iteration. For each iteration, many of these cells are likely to be scattered in the scene. Consequently, these cells may not be mutually visible. The selection of one of them would imply a large disk transfer cost, which increases the running time. On the other hand, if the transfer costs are evaluated in terms of number of cells, a cell can be selected as shooting cell and then loaded into memory, not necessarily at the end of the iteration, even though it contains many surfaces. Thus, expressing transfer costs in terms of number of cells is more efficient because spatial coherence is fully exploited.

So far, we have experimented with our ordering strategies for a few complex scenes because of the very important computing time needed. From these experiments, we have ascertained that the traveling salesman algorithm always provides the best results. This can be explained by the fact that in a preprocessing step this algorithm predicts, at one go, all the N successive best shooting cells, N being the total number of cells, even though it does not provide an optimal solution since it accounts for only read operations as explained previously (see section 4.2.4). Moreover, the cost of this preprocessing is insignificant, which makes this algorithm outperform the backtracking strategy.

Regarding the backtracking algorithm, it provides an optimal solution (i.e. lowest number of disk transfers) by predicting at one go a limited number n ($n \ll N$) of best successive shooting cells while accounting for read and write operations. The repeated predictions are very time consuming which dramatically affects the performances of this algorithm.

Figure 3 shows an image of the building obtained with our method after complete resolution.

6 Conclusion

We have proposed seven ordering strategies for computing radiosity solutions in large environments. They are

| Random Method | Greedy S | Greedy C | Traveling Salesman | Backtracking S | Backtracking C | Max Energy |
|---------------|-----------|----------|--------------------|----------------|----------------|------------|
| 112h 54mn | 100h 46mn | 79h 16mn | 56h 30mn | 108h 36mn | 97h 12mn | 132h 34mn |

Table 1. Computing time for the seven ordering strategies.

| Random Method | Greedy S | Greedy C | Traveling Salesman | Backtracking S | Backtracking C | Max Energy |
|---------------|----------|----------|--------------------|----------------|----------------|------------|
| 538,698 | 348,584 | 246,012 | 452,176 | 312,434 | 183,924 | 375,148 |

Table 2. Amount of disk transfers expressed int terms of input surfaces.

based on the partitioning of the scene into cells and on simple visibility calculations between these cells. Only a subset of cells need to reside in memory: the shooting cell and those visible from it. These resident cells change during the resolution process, implying then many read and write operations from or onto the disk. The role of these strategies is to reduce the number of these disk transfers. It turns out that the best strategy is Traveling Salesman. Further experiments are needed to draw definitive conclusions on these strategies.

Unlike the methods described in [1, 6] our algorithm uses shooting rather than gathering so as to offer to the user the possibility of interactively rendering the scene (at the intermediate stages of the resolution) while pursuing the radiosity computation.

Another advantage of our method is its easy adaptation to the available computing resources. Indeed, it is possible to stop the calculations after a certain number of *iterations* and to restart them later on without affecting the final radiosity solution. Moreover, since our method manages the memory space where are stored the cells involved in the current radiosity computation, we can set M_{limit} to a value depending on the size of the main memory available on the used computer.

We are currently investigating other ordering strategies accounting for other criteria: energy distribution, visibility graph, etc.

Finally, these ordering algorithms are the starting point for devising a parallel version. A processor could be in charge of a shooting cell and those visible to it, which will ensure data locality. This latter would increase the performances of the parallel algorithm.

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```

global_illumination() {
    typedef struct CELL_FLAGS {
        boolean In_Memory;
        boolean To_Be_Used;
    };

    Integer C;          /* Cell identifier */
    CELL_FLAGS Cell_In_Memory[number_of_cells];
    ITERATION = 0;
    /* VG is the visibility graph */
    VG = Read_Visibility_Graph_From_Disk();
    Initialize(Cell_In_Memory[ ]);
    While not (convergence) AND (ITERATION < Max_Iterations) {
        Unmark_All_The_Cells();
        /* Choose the first shooting cell */
        C = Choose_Cell(VG, ITERATION, Cell_In_Memory);
        /* Choose_Cell() modifies Cell_In_Memory[ ] */
        Mark_Cell(C);
        /* The cell C is marked during the current iteration */
        while not (all the cells are marked) {
            /* The cell C shoots now its energy */
            Shoot_From_Cell(C, Cell_In_Memory);
            /* Choose a shooting cell which has not already */
            /* shot its energy during this iteration */
            C = Choose_Cell(VG, ITERATION, Cell_In_Memory);
            Mark_Cell(C);
        }
        ITERATION = ITERATION + 1;
    }
}

```

Figure 1. Global algorithm

```

int Choose_Cell(VG,ITERATION,Cell_In_Memory)
{
    int cost_min, cost;
    cost_min = HUGE;
    for each cell  $C_i$  {
        /* Find the set VS of cells visible from  $C_i$  */
        VS = find_visible_Set( $C_i$ ,VG);
        cost = Evaluate_Cost( $C_i$ ,VS,Cell_In_Memory)
        if ( cost < cost_min ) {
            cost_min = cost;
            result =  $C_i$ ;
        }
    }
    Update(VG, $C_i$ ,Cell_In_Memory);
    return result;
}

```

Figure 2. General ordering algorithm



Figure 3. Image of the total scene once the complete global illumination has been performed.

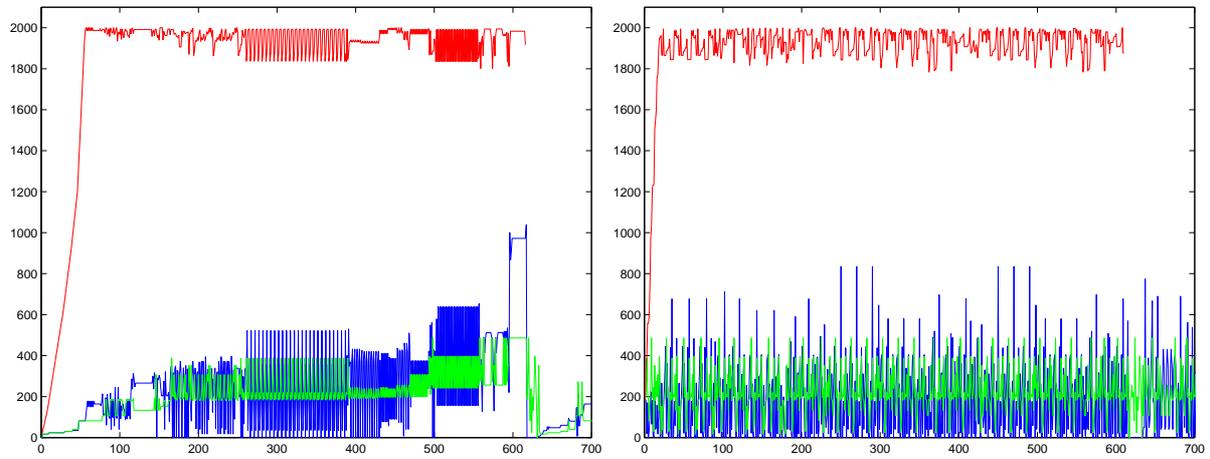


Figure 4. Results in terms of number of surfaces. Upper image: Greedy S. Lower image: Greedy C

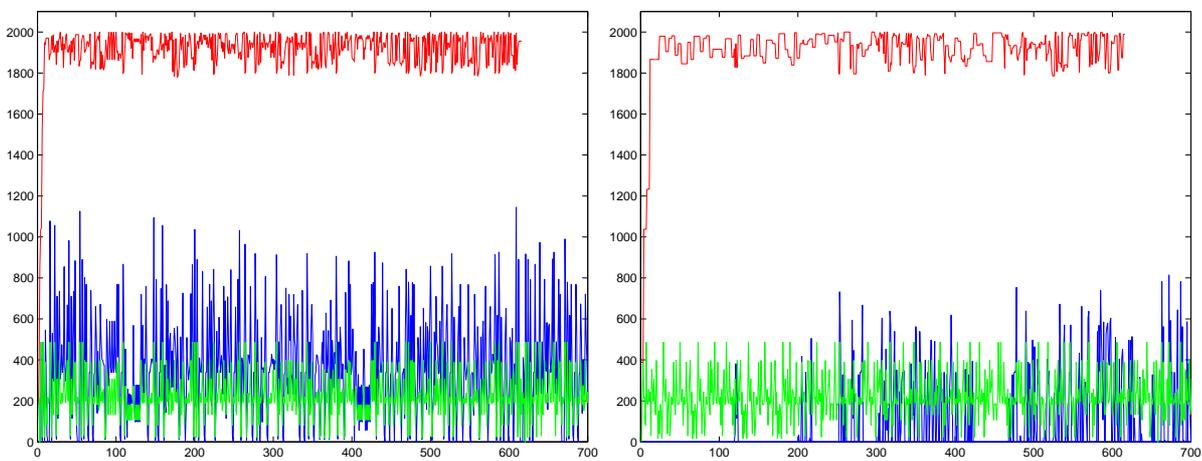


Figure 5. Results in terms of number of surfaces. Upper image: Traveling Salesman. Lower image: Backtracking C